Theorem

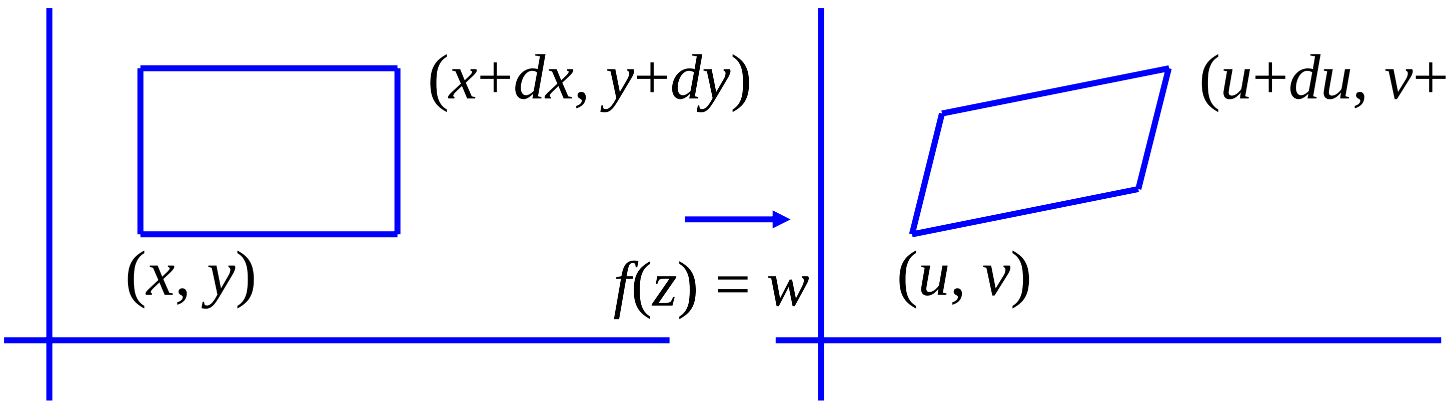
If a function $f(z)=u+iv$ is analytic in D, then the Cauchy-Rieman equations hold, i.e.

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

and $$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Alternatively, if the Cauchy-Rieman equations hold for a function $f(z)=u+iv$ and the function has continuous first order partial derivatives, then the $f(z)$ is analytic in D.

Cauchy Riemann Eqns



$$f'(z) = \lim_{dz \to 0} [f(z+dz) - f(z)]/dz = \lim_{dx,\, dy \to 0} [(u+du)+i(v+dv)-(u+iv)]/[dx+idy]$$

$$= \lim_{dx,\, dy \to 0} [du + idv]/[dx + idy] = \partial u/\partial x + i\partial v/\partial x \quad \text{if } dy = 0$$

$$= \partial v/\partial y - i\partial u/\partial y \quad \text{if } dx = 0$$

CR equations follow.