NUS ECE Department EE2012 Analytical Methods in ECE Mid-term Test

Time Allowed: 50++ min **Max Marks: 105**

Do all seven (7) questions, each is worth 15 points. Mark your answers on the sheet. Each question has several statements. Circle the appropriate letter for the True/False type choices.

Question 1.

A) A sample of size one is too small to be an unbiased estimator of the population mean. **T or F**
(4 marks)

B) Any estimator can be unbiased if it is based on a very large sample size. **T or F**
(4 marks)

C) A consistent estimator is also an unbiased estimator but the reverse is in general not true. **T or F**
(4 marks)

D) $\frac{1}{n-1}\left(X_1 + X_2 + \ldots X_n\right)$ is an unbiased and consistent estimator. **T or F**
(3 marks)

Question 2.

A) A WSS process is also SSS if its mean is time-invariant. **T or F**
(4 marks)

B) Except for a Gaussian random process, an SSS process is also WSS. **T or F**
(4 marks)

C) The variance of a zero mean WSS process is equal to the autocorrelation function at $\tau = 0$. **T or F**
(4 marks)

D) If $\mathbf{X}(t) = A\cos wt + B\sin wt$ is a WSS process, where A and B are random variables, then the mean of A and B must be zero. **T or F**
(3 marks)

Question 3.

A) If $\mathbf{X}(t)$ is constant, then $R_X(t)$ is also constant. **T or F**
(4 marks)

B) For two jointly WSS random processes $\mathbf{X}(t)$ and $\mathbf{Y}(t)$, $R_{YX}(\tau) = -R_{XY}(-\tau)$. **T or F**
(4 marks)

C) If $\mathbf{X}(t)$ and $\mathbf{Y}(t)$ are independent WSS random processes with mean μ_X and μ_Y, respectively, then $R_{XY}(t, t+\tau) = \mu_X \mu_Y$. **T or F**
(4 marks)

D) Let $\mathbf{X}(t)$ be the input and $\mathbf{Y}(t)$ the output of a linear time-invariant system. A zero mean $\mathbf{X}(t)$ can produce a non zero mean $\mathbf{Y}(t)$. **T or F**
(3 marks)

Question 4.

A) $S_X(\omega)$ is real and can be negative or positive depending on the process **X**(t). **T or F**
(4 marks)

B) $S_X(\omega) = S_X(-\omega)$. **T or F**
(4 marks)

C) There exists a process **X**(t) with power spectral density $S_X(\omega) = \sin \omega / \omega$. **T or F**
(4 marks)

D) Show that the variance of a zero mean WSS process is equal to $\frac{1}{2\pi}\int_{-\infty}^{+\infty} S_x(\omega)d\omega$.

Solution: Since $R_X(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_x(\omega)e^{j\omega\tau}d\omega$ and $R_X(0)=Var[X(t)]$ if $X(t)$ is zero mean, we obtain $Var[X(t)] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_x(\omega)d\omega.$ (3 marks)

Question 5.

In a biomedical lab, a drug has a mean cure rate of 0.4. It was tested on 500 mice with a certain disease. Based on hypothesis testing at 0.05 level of significance, it is now claimed that the mean cure rate $\mu > 0.4$.

A) The hypotheses are H_0: $\mu = 0.4$ versus H_1: $\mu > 0.4$. **T or F**
(4 marks)

B) The hypotheses are H_0: $\mu = 0.4$ versus H_1: $\mu < 0.4$. **T or F**
(4 marks)

C) Maximum likelihood estimate of mean cure rate is given by $\frac{1}{n}(X_1 + X_2 + \ldots X_n)$.
T or F
(4 marks)

D) Maximum likelihood estimate of mean cure rate is always an efficient estimator.
T or F
(3 marks)

Question 6.

A) Type I error occurs when the alternative hypothesis is accepted when it is invalid.
T or F
(4 marks)

B) Type I error occurs when the alternative hypothesis is rejected when it is valid.
T or F
(4 marks)

C) Type II error occurs when the alternative hypothesis is rejected when it is valid.
T or F
(4 marks)

D) One can reduce Type I and Type II errors simultaneously. **T or F**
(3 marks)

Question 7.

A) In interval estimation, the larger the sample size, the smaller the width of the interval would be, for a fixed level of confidence. **T or F**
(4 marks)

B) A preference test based on interval estimation is used to establish whether Singaporeans prefer chicken rice or laksa. Based on a sample of size 1000, it is found that the proportion of Singaporeans preferring chicken rice lies in the interval (0.25, 0.51). Therefore, we can conclude that Singaporeans prefer laksa. **T or F**
(4 marks)

C) In interval estimation, the larger the level of confidence, the smaller the width of the interval would be, for a fixed sample size. **T or F**
(4 marks)

D) Derive the smallest sample size required for interval estimation based on Z under level of significance α and margin of error $\pm E$.

Solution: From $\overline{X} - \frac{\sigma}{\sqrt{n}} z_{\alpha/2} < \mu < \overline{X} + \frac{\sigma}{\sqrt{n}} z_{\alpha/2}$, we obtain $E = \frac{\sigma}{\sqrt{n}} z_{\alpha/2}$. Thus, the smallest sample size needed is $\left[\frac{\sigma_{\max}}{E} z_{\alpha/2} \right]^2$, where $\sigma_{\max}$ is the maximum possible value of σ.
(3 marks)