Matriculation Number: ___________

NUS ECE Department EE2012 Analytical Methods in ECE Mid-term Test

Time Allowed: 50++ min **Max Marks: 105**

Answer any SEVEN questions. Mark your answers on the sheet. Each question has several statements. Circle the appropriate letter for the True/False type choices.

Question 1. A test consists of 10 multiple-choice questions. Each question has 4 choices. A student who did not study for the test attempted each question by picking a choice in a random manner.

A) What is the probability of incorrectly answering all the questions?

ANS: $\left(\frac{3}{4}\right)^{10}$ (4 marks)

B) What is the probability of correctly answering all the questions?

ANS: $\left(\frac{1}{4}\right)^{10}$ (4 marks)

C) Write the probability expression of answering correctly more than 6 questions.

ANS: $\sum_{k=7}^{10}\binom{10}{k}\left(\frac{1}{4}\right)^{k}\left(\frac{3}{4}\right)^{10-k}$ (4 marks)

D) Write the probability expression of answering correctly between 2 to 8 questions.

ANS: $\sum_{k=2}^{8}\binom{10}{k}\left(\frac{1}{4}\right)^{k}\left(\frac{3}{4}\right)^{10-k}$ (3 marks)

Question 2. In a bio-medical lab, four bio-experiments are modelled by random variables X_1, X_2, X_3 and X_4. Their means and variances are given in the following table.

Random Variable	**Mean**	**Variance**
X_1	5	0.5
X_2	6	0.2
X_3	2	0.8
X_4	8	0.1

Further it is given that only X_1 and X_3 are SI.

A) $E[X_1 - X_3] = 7$ **T or F** (4 marks)

B) $V[X_1 + X_2] = 1.0$ **T or F**

ANS: $V[X_1 + X_2] = V[X_1] + V[X_2] + 2\rho\sqrt{V[X_1]V[X_2]}$

$$= 0.5 + 0.2 + 2\rho\sqrt{(0.5)(0.2)}$$

Since $-1 \le \rho \le 1$, it follows that $0.007 \le V[X_1 + X_2] \le 1.332$.

So can be true and can be false! (4 marks)

C) $V[X_3 - X_1] = 0.3$ **T or F** (4 marks)

D) Find $E[X_1^2 - X_2^2 + X_3^2 - X_4^2]$.

ANS: $[0.5+5^2]-[0.2+6^2]+[0.8+2^2]-[0.1+8^2]=-70$ (3 marks)

Question 3.

A) $E[XY] = E[X]E[Y]$ implies that *X* and *Y* are SI. **T or F** (4 marks)

B) If *X* and *Y* are uncorrelated then they are also SI. **T or F** (4 marks)

C) If *X* and *Y* are SI then they are also uncorrelated. This statement is only true for Gaussian random variables. **T or F** (4 marks)

D) Orthogonal random variables must also be independent. **T or F** (3 marks)

Question 4. Let $X_1, X_2, \ldots, X_N$ be random variables. Define $X = X_1 + X_2 + \ldots + X_N$.

A) If *N* is large and $X_1, X_2, \ldots, X_N$ are uncorrelated then *X* tends to a Gaussian random variable. **T or F** (4 marks)

B) If $X_1, X_2, \ldots, X_N$ are Gaussian random variables then *X* tends to a Gaussian random variable only if *N* is large. **T or F** (4 marks)

C) If *N* is large and $X_1, X_2, \ldots, X_N$ are SI then *X* tends to a Gaussian random variable only if the PDFs of $X_1, X_2, \ldots, X_N$ are the same. **T or F** (4 marks)

D) If $Y = X_1^{88} + X_2^{88} + \ldots + X_N^{88}$ then *Y* tends to a Gaussian random variable if *N* is large and $X_1, X_2, \ldots, X_N$ are SI. **T or F** (3 marks)

Question 5. Let *X* be a Gaussian random variable with mean 0.9 and variance 0.01. Given that $P(0.7 < X < 0.9) = 0.3$ and $P(0.9 < X < 1.2) = 0.4$. Then

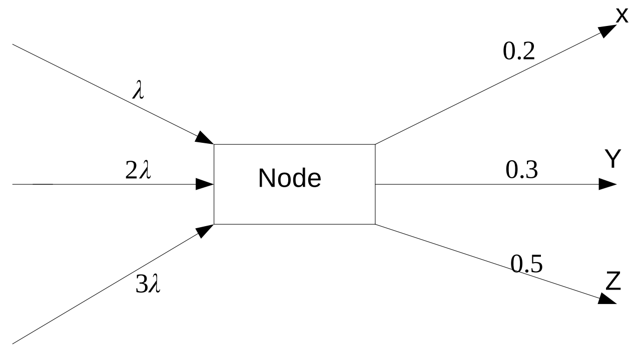
A) $P(X > 0.7) = 0.8$ **T or F** (4 marks)

B) $P(X > 1.5) > 0.1$ **T or F** (4 marks)

C) $P(X < 0.7) > 0.4$ **T or F** (4 marks)

D) $P(0.7 < X < 1.2) = 0.7$ **T or F** (3 marks)

Question 6. Data packets flow to a computer node N in three paths according to Poisson processes with rates as indicated in figure below. The packets are then routed to points X, Y and Z with probabilities 0.2, 0.3 and 0.5, respectively.



A) Find the probability of getting k packets in X.

ANS: $\dfrac{(1.2\lambda)^k}{k!}\exp(-1.2\lambda)$ (4 marks)

B) Find the probability of getting k packets in Y.

ANS: $\dfrac{(1.8\lambda)^k}{k!}\exp(-1.8\lambda)$ (4 marks)

C) Find the probability of getting k packets in Z.

ANS: $\dfrac{(3\lambda)^k}{k!}\exp(-3\lambda)$ (4 marks)

D) Find the probability of getting k packets in the node N.

ANS: $\dfrac{(6\lambda)^k}{k!}\exp(-6\lambda)$ (3 marks)

Question 7.

A) $P(A \cap B) = P(A)P(B)$ if A and B are mutually exclusive events. **T or F** (4 marks)

B) $P(A \mid B) = P(A)$ if A and B are independent events. **T or F** (4 marks)

C) $P(A \cup B) = P(A) + P(B)$ if A and B are independent events. **T or F** (4 marks)

D) $P(A \mid B)P(B) = P(B \mid A)P(A)$ if A and B are independent events. **Tor F** (3 marks)

Question 8. Let $g(x)$ and $h(y)$ be the pdfs of random variables X and Y, respectively, and $f(x, y)$ be their joint pdf. Let A be a constant.

A) $f(x, y) = Axy,\ 0 < x < 1, 0 < y < 1\ \Rightarrow X$ and Y are SI. **T or F** (4 marks)

B) $f(x, y) = Ae^{-x}e^{-y},\ 0 < y < x,\ 0 < x < 1 \Rightarrow X$ and Y are SI. **T or F** (4 marks)

C) $f(x, y) = A\sqrt{x+y},\ 0 < x < 1,\ 0 < y < 1\ \Rightarrow X$ and Y are SI. **T or F** (4 marks)

D) $f(x, y) = g(x)h(y) \Rightarrow X$ and Y are SI. **T or F** (3 marks)

Question 9. The lifetime of a light bulb is modelled as a random variable X with cdf

$$F(t) = \begin{cases} 1 - \exp(-\lambda t) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

A) Find the pdf of X. ANS: $\lambda \exp(-\lambda t)$ (4 marks)

B) Find $P(T > x)$. ANS: $\exp(-\lambda x)$ (4 marks)

C) Find $P(T - t > x \mid T > t)$. ANS : $\exp(-\lambda x)$ (4 marks)

D) Show that $P(T - t > x \mid T > t) = P(T > x)$. ANS: By comparing results in B and C. (3 marks)

Question 10. In a telecommunication network, the transmitted signal S and received signal R are related as

$$R = S + N$$

where N is the noise in the channel modelled as a random variable having uniform pdf in the interval [-1.1 1.1]. Given that S is either 1 Volt or 0 Volt, and both are transmitted with equal probability.

A) $E(S) = 0$. **T or F** (4 marks)

B) The mean of R given that $S = 1$ is the same as mean of R given that $S = 0$. **T or F** (4 marks)

C) By central limit theorem, pdf of R is approximately Gaussian. **T or F** (4 marks)

D) Express the pdf of R.

ANS: $f(r) = 0.5 f(r \mid s = 1) + 0.5 f(r \mid s = 0)$ where

$$f(r \mid s = 1) = \begin{cases} 1/2.2 & r \in (-0.1,\ 2.1) \\ 0 & otherwise \end{cases} \quad \text{and} \quad f(r \mid s = 1) = \begin{cases} 1/2.2 & r \in (-1.1,\ 1.1) \\ 0 & otherwise \end{cases}$$

(3 marks)