Matric Number:___________

NUS, ECE Department; EE2012 Analytical Methods in ECE: Mid-term Test

Time Allowed: 50++ min **Max Marks: 100**

Answer all questions. Mark your answers on the sheet. Each question has several statements. Circle the appropriate letter for the True/False type choices.

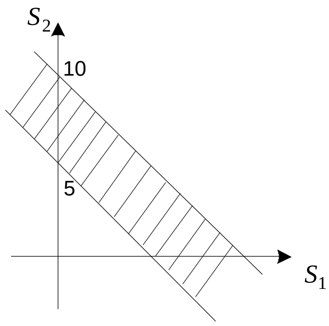
1. Let $S_1 \sim N(\mu_1, \sigma_1^2)$ and $S_2 \sim N(\mu_2, \sigma_2^2)$ be two signals having Gaussian pdf. Assume that S_1 and S_2 are not independent.

A. $S_1 + S_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$ **T or F** **(2 marks)**

B. $S_1 + S_1 + S_1 + S_1 + S_1 \sim N(5\mu_1, 5\sigma_1^2)$ **T or F** **(2 marks)**

C. Since S_1 and S_2 are not independent $S_1 + S_2$ is not a Gaussian rv. **T or F** **(2 marks)**

D. Mark the region corresponding to the event A: $5 < S_1 + S_2 < 10$.



(5 marks)

2. A manufacturing process is modeled by a binomial rv X with parameters (N, 0.3). Another process is modeled by a binomial rv Y with parameters (M, 0.2). Assume X and Y are independent.

A. $X+Y$ is also a binomial random variable. **T or F** **(2marks)**

B. X is formed from N Bernoulli trials. **T or F** **(2 marks)**

C. If M is sufficiently large, then $X + Y$ can be approximated by a Gaussian random variable. **T or F** **(2 marks)**

D. Write an exact expression for P($X + Y$ < 10).

$$P(X+Y<10) = \sum_{k=0}^{9} P(X=k)P(Y \le 9-k)$$

$$= \sum_{k=0}^{9}\left[\frac{N!}{k!(N-k)!}0.3^k 0.7^{N-k} \sum_{j=0}^{9-k} \frac{M!}{j!(M-j)!}0.2^j 0.8^{M-j}\right]$$

(5 marks)

3. The number of data packets arriving at a node A is modeled by Poisson rv X:

$$P(X = k) = e^{-10,000} 10,000^k / k!, \;\; k = 0,1,\ldots$$

A. P(X > 50) is close to 1. **T or F** **(2 marks)**
B. P(X > 50) is close to 0. **T or F** **(2 marks)**
C. We can find P(X > 500) by approximating X by a Gaussian random variable. **T or F** **(2 marks)**
D. A gets congested if X exceeds 4000. Write an expression for P(A).

$$P(A) = P(X > 4000) = 1 - P(X \le 4000) = 1 - \sum_{k=0}^{4000} e^{-10,000} 10,000^k / k!$$

(5 marks)

4. A series of N uncorrelated signals from a satellite are represented by the Gaussian random variables $X_1, X_2, \ldots X_N$.

A. $X_1 + X_2 + \ldots + X_N$ is Gaussian only if N is sufficiently large. **T or F** **(2 marks)**
B. $X_1, X_2, \ldots X_N$ may or may not be statistically independent. **T or F** **(2 marks)**
C. The joint PDF of $X_1, X_2, \ldots X_N$ is not necessary the product of the individual marginal PDFs of $X_1, X_2, \ldots X_N$. **T or F** **(2 marks)**
D. Find the pdf of $X_1 - X_2 + \ldots + (-1)^{N-1} X_N$.

Let $X_i \sim N(\mu_i, \sigma_i^2)$ for $i = 1,2,\ldots,N$.
$X_1, X_2, \ldots X_N$ are uncorrelated
$\Rightarrow X_1, X_2, \ldots X_N$ are statistically independent
$\Rightarrow X_1 - X_2 + \ldots + (-1)^{N-1} X_N \sim N(\mu, \sigma^2)$, where
$\mu = \mu_1 - \mu_2 + \ldots (-1)^{N-1} \mu_N$ and $\sigma^2 = \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_N^2$.

Therefore, pdf of is $X_1 - X_2 + \ldots + (-1)^{N-1} X_N$ is $\dfrac{1}{\sqrt{2\pi}\sigma} \exp\left[-\dfrac{(x-\mu)^2}{2\sigma^2} \right]$.

(5 marks)

5. A mice tested with a new drug will experience side effect X with probability 0.2, side effect Y with probability 0.4, and both side effects X and Y with probability 0.08.

A. The side effect X and side effect Y are independent. **T or F** **(2 marks)**
B. The side effect X and side effect Y are mutually exclusive. **T or F** **(2 marks)**
C. Can't tell whether the side effects X and Y are independent. **T or F** **(2 marks)**

D. Can't tell whether the side effects X and Y are mutually exclusive. **T or F** **(2 marks)**

6. Each year the stock market crashes with probability 0.1. It crashed in year 0. Let X be a rv denoting number of years that pass before two consecutive crashes.

A. X is a Poisson rv modeling the "arrival" of the crash. **T or F** **(2marks)**

B. X is a geometric rv. **T or F** **(2 marks)**

C. X is a binomial rv. **T or F** **(2 marks)**

D. Write an exact expression for P(X = 3).

P(X = 3) = P(Crash in year 1,4,5) + P(Crash in year 2,4,5) + P(Crash in year 4,5) $= (0.1)^3(0.9)^2 + (0.1)^3(0.9) + (0.1)^2(0.9)^3$.

(5 marks)

7. Time of birth is noted as hour:minute:second, eg, 08:30:12 or 22:40:03. In a class of 300 students, probability that two students have same time of birth is

A. close to 1 **T or F** **(2 marks)**

B. close to 0.75 **T or F** **(2 marks)**

C. close to 0.25 **T or F** **(2 marks)**

D. close to 0 **T or F** **(2 marks)**

Prob(two students have same time of birth)

=1 – prob(no students have same time of birth)

$$1-\frac{86400(86400-1)\cdots(86400-299)}{86400^{300}} < 1-\frac{(86400-299)^{300}}{86400^{300}} = 0.6465$$

The actual answer is 0.4053, which is somewhat amazing considering there are 86400 seconds in a day and only 300 students. Due to time limitation, it is difficult for students to arrive at the actual answer. So if **F** is circled for A and D and **T** is circled for B and C full marks will be given. The students may also check **T** for just one of B and C & **F** for all others and still collect full credit.

8. Let X be a Gaussian random variable with mean 0.5. If $P(0.1 < X < 0.5) = 0.3$ and $P(0.5 < X < 1.0) = 0.4$, then

A. $P(X < 0.1) = 0.2$ **T or F** **(2 marks)**

B. $P(X > 1.0) = 0.1$ **T or F** **(2 marks)**

C. $P(0.1 < X < 1.0) = 0.7$ **T or F** **(2 marks)**

D. Compute $P(X^2 < 1)$ approximately.

$P(X^2 < 1) = P(-1 < X < 1) \approx P(X < 1) = 0.9.$ **(5 marks)**

9. Let X and Y be two orthogonal rv.

A. The mean of XY is zero. **T or F** **(2 marks)**

B. If mean of X and mean of Y are zero, then X and Y are also uncorrelated. **T or F** **(2 marks)**

C. If X and Y are uncorrelated, then they are also statistically independent. **T or F** **(2 marks)**

D. If X and Y are jointly Gaussian, they are statistically independent. **T or F** **(2 marks)**

10. Let X and Y be two rv with marginal PDFs $f(x)$ and $h(y)$, respectively. Their joint PDF is denoted by $f(x, y)$.

A. If X and Y are independent, then $MGF[X+Y] = MGF[X] + MGF[Y]$. **T or F** **(2 marks)**

B. If $E[XY] = E[X]E[Y]$, then X and Y are independent. **T or F** **(2 marks)**

C. If X and Y are uncorrelated Gaussian rv, then $f(x, y) = f(x)h(y)$. **T or F** **(2 marks)**

D. If X and Y are independent, then write the pdf of $X + Y^3$ as a convolution of two pdfs.

X and Y are independent $\Rightarrow$ X and Y^3 are also independent.
Let $Z=Y^3$ and pdf of Z be $g(z)$. Then pdf of $W=X+Y^3$ is the convolution of $f(x)$ and $g(z)$ given as

$$\int_{-\infty}^{\infty} f(u)g(w-u)du.$$ **(5 marks)**