More Sample Mid-Term Test Questions

Question 1.

A test consists of N multiple-choice questions. Each question has 4 choices. A student who did not study for the test attempted each question by picking a choice in a random manner.

A) What is the probability of incorrectly answering all the questions?

ANS: $\left(\frac{3}{4}\right)^N$

B) What is the probability of correctly answering all the questions?

ANS: $\left(\frac{1}{4}\right)^N$

C) Write the probability expression of answering correctly more than 6 questions.

ANS: $\sum_{k=7}^{N}\binom{N}{k}\left(\frac{1}{4}\right)^k\left(\frac{3}{4}\right)^{N-k}$

D) Write the probability expression of answering correctly between 2 to 8 questions.

ANS: $\sum_{k=2}^{8}\binom{N}{k}\left(\frac{1}{4}\right)^k\left(\frac{3}{4}\right)^{N-k}$

Question 2.

In a bio-medical lab, four bio-experiments are modelled by random variables X_1, X_2, X_3 and X_4. Their means and variances are given in the following table.

Random Variable	**Mean**	**Variance**
X_1	5	0.5
X_2	6	0.2
X_3	2	0.8
X_4	8	0.1

Further it is given that only X_1 and X_3 are SI.

A). $E[X_1 - X_3] = 3$ **T or F**

B) $V[X_1 + 2X_3] = 3.7$ **T or F**

C). $V[aX_3 - X_1] = 0.8a$ -0.5 **T or F**

D) Find $E[2X_1^2 + 3X_2^2 + 4X_3^2 + 5X_4^2]$.

ANS: $2[0.5+5^2]+3[0.2+6^2]+4[0.8+2^2]+5[0.1+8^2]$

Question 3.

A) $E[XY] = E[X] + E[Y]$ implies that X and Y are SI. **T or F**

B) If X and Y are uncorrelated then they are also orthogonal **T or F**

C) If X and Y are SI then they are also uncorrelated. **T or F**

D) Orthogonal random variables must have zero mean. **T or F**

Question 4.

Let $X_1, X_2, \ldots, X_N$ be random variables. Define $X = X_1 + X_2 + \ldots + X_N$.

A) If N is large and $X_1, X_2, \ldots, X_N$ are orthogonal to each other then X tends to a Gaussian random variable. **T or F**

B) If $X_1, X_2, \ldots, X_N$ are SI binomial random variables then X is also binomial. **T or F**

C) If N is large and $X_1, X_2, \ldots, X_N$ are SI then X tends to a Gaussian random variable even if the PDFs of $X_1, X_2, \ldots, X_N$ are different. **T or**

D) If $Y = \ln X_1 + \ln X_2 + \ldots + \ln X_N$ then Y tends to a Gaussian random variable if N is large and $X_1, X_2, \ldots, X_N$ are SI. **T or F**

Question 5.

Let X be a Gaussian random variable with mean 0.9 and variance 0.1. Given that $P(0.7 < X < 0.9) = 0.3$ and $P(0.9 < X < 1.2) = 0.4$. Then

A) $P(X > 0.7) = 0.8$ **T or F**

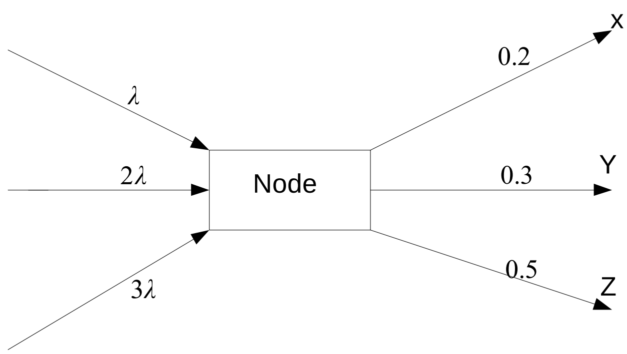
B) $P(X > 1.5) < 0.1$ **T or F**

C) $P(X < 0.7) < 0.4$ **T or F**

D) $P(0.7 < X < 1.2) = 0.9$ **T or F**

Question 6.

Data packets flow to a computer node N in three paths according to Poisson processes with rates as indicated in figure below. The packets are then routed to points X, Y and Z with probabilities 0.2, 0.3 and 0.5, respectively.



A) Find the probability of getting k packets in X.

ANS: $\dfrac{(1.2\lambda)^k}{k!}\exp(-1.2\lambda)$

B) Find the probability of getting k packets in Y.

ANS: $\dfrac{(1.8\lambda)^k}{k!}\exp(-1.8\lambda)$

C) Find the probability of getting k packets in Z.

ANS: $\dfrac{(3\lambda)^k}{k!}\exp(-3\lambda)$

D) Find the probability of getting k packets in the node N.

ANS: $\dfrac{(6\lambda)^k}{k!}\exp(-6\lambda)$

Question 7.

A) $P(A\cap B)=P(A)+P(B)$ if A and B are mutually exclusive events. **T or F**

B) $P(A\mid B)=P(A)$ if A and B are independent events. **T or F**

C) $P(A\cup B)=P(A)P(B)$ implies A and B are independent events. **T or F**

D) $P(A\mid B)P(B)=P(A\cup B)$ if A and B are independent events. **Tor F**

Question 8.

Let $g(x)$ and $h(y)$ be the PDFs of random variables X and Y, respectively, and $f(x, y)$ be their joint PDF. Let A be a constant.

A) $f(x,y)=Axy,\ 0<x<y,\ 0<y<1\ \Rightarrow X$ and Y are SI. **T or F**

B) $f(x,y)=Ae^{-x}e^{-y},\ 0<y<1,\ 0<x<1\Rightarrow X$ and Y are SI. **T or F**

C) $f(x,y)=A\sqrt{x+y},\ 0<x<1,\ 0<y<1\ \Rightarrow X$ and Y are uncorrelated **T or F**

D) $f(x,y)=g(x)$ convolve $h(y)\Rightarrow X$ and Y are SI. **T or F**

Question 9.

The lifetime of a light bulb is modelled as a random variable X with

$$P(X>x)=\begin{cases}\exp(-\lambda x) & x\ge 0\\ 0 & x<0\end{cases}$$

A) Find the PDF of X. ANS: $\lambda\exp(-\lambda x)$

B) Find $P(X<x)$. ANS: $1-\exp(-\lambda x)$

C) Find $P(X-t>x\mid X>t)$. ANS : $\exp(-\lambda x)$

D) Show that $P(T-t>x\mid T>t)=P(T>x)$. ANS: By using results in B and C.

Question 10.
In a telecommunication network, the transmitted signal S and received signal R are related as

$$R = S + N$$

where N is the noise in the channel modelled as a random variable having uniform PDF in the interval [-1.1 1.1]. Given that S is either 1 Volt or 0 Volt, and both are transmitted with equal probability.

A) $E(S) = 1/2$. **T or F**
B) The mean of R given that $S = 1$ is the same as mean of N. **T or F**
C) The PDF of R when S=-1 is a uniform PDF. **T or F**
D) Express the PDF of $f(r|s=1)$.

ANS: $f(r \mid s=1) = \begin{cases} 1/2.2 & r \in (-0.1,\, 2.1) \\ 0 & otherwise \end{cases}$

Question 11.
A bio-experiment consists of a sequence of SI trials. Each trial has 3 outcomes H, T and E. At the i-th trial, $i = 1,2,\ldots$, $P(H) = 0.2$, $P(T) = 0.3 + 1/(10i)$ and $P(E) = 0.5 - 1/(10i)$.

A) The trials can be classified as Bernoulli trials. **T or F**
B) The trials can be termed as Bernoulli trials if we are interested only in the occurrence of H. **T or F**
C) The trials can be termed as Bernoulli trials if we are interested only in the occurrence of T. **T or F**
D) If 100 trials are performed, write an expression for the probability of getting 50 H.

$$\text{ANS}: \quad P(50H) = \frac{100!}{50!50!} 0.2^{50} 0.8^{50}$$

Question 12.
Let $X_1, X_2, \ldots$ be a sequence of Gaussian RVs; $Y_1, Y_2, \ldots$ a sequence of binomial RVs with $N = 10$ and $p = 0.2$; and $Z_1, Z_2, \ldots$ a sequence of binomial RVs with $N = 20$ and $p = 0.2$. All RVs are SI.

A) $X_1 + X_2 + \ldots + X_M$ is Gaussian **T or F**
B) $(Y_1 + Z_1) + (Y_2 + Z_2) + \ldots + (Y_M + Z_M)$ is binomial. **T or F**
C) $Z_1 + Z_2 + \ldots + Z_M$ has mean $0.2N$ **T or F**
D) $\sum_{i=1}^{M} \exp(X_i + Y_i + Z_i)$ is Gaussian when M is large. **T or F**

Question 13.
Let $h(x)$ and $g(y)$ be marginal PDF of X and Y, respectively, and $f(x, y)$ be the joint PDF of (X, Y).

A) $f(x, y) = g(x)h(y)$ implies $E(XY) = E(X)\,E(Y)$ if X and Y are Gaussian **T or F**
B) If X and Y are uncorrelated RVs then we must have $f(x, y) = g(x)h(y)$. **T or F**
C) If X and Y are SI then $P(X<x, Y<y) = P(X<x)P(Y<y)$. **T or F**
D) If X and Y are SI, then they must be orthogonal. **T or F**

Question 14. Let $X \sim N(0.1, 0.05)$ and $Y \sim N(0.2, 0.05)$. In addition, $\mathrm{cov}(X, Y) = 0.03$.

A) $V(X + Y) = 0.16$. **T or F**
B) $\mathrm{MGF}(X + Y)=\mathrm{MGF}(X)\mathrm{MGF}(Y)$ **T or F**
C) $P(X + Y < 0.3) = 0.5$. **T or F**
D) $E(2X + 3Y) = 0.33$. **T or F**

Question 15.
A) A sample of size one is an unbiased estimator of the population mean. **T or F**
B) An unbiased estimator is a consistent estimator. **T or F**
C) An efficient estimator is also an unbiased estimator. **T or F**
D) ML estimator can be a biased estimator. **T orF**

Question 16.
A) $P(A \mid B)P(B) = 0$ if A and B are mutually exclusive events. **T or F**
B) $P(A \mid B) = P(B \mid A)$ implies A and B are mutually exclusive events. **T or F**
C) $P(A \cup B) - P(A) - P(B) = 0$ if A and B are SI events. **T or F**
D) $P(A \mid B)P(B) = P(A \cap B)$ if A and B are SI events. **T or F**

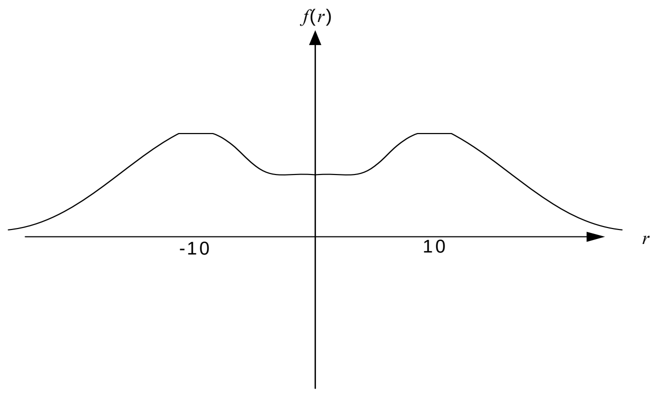
Question 17. Let $g(x)$ and $h(y)$ be the PDFs of RVs X and Y, respectively, and $f(x, y)$ be their joint PDF. Let A be a constant.

A) $f(x, y) = Ax^2y^3,\ 0 < x < 1,\ 0 < y < x \ \Rightarrow X$ and Y are SI. **T or F**
B) $f(x, y) = Ae^{-(x^2+y^2)},\ 0 < y < 1,\ 0 < x < 1 \Rightarrow X$ and Y are SI. **T or F**
C) $f(x, y) = A\ln(x + y),\ 0 < x < 1,\ 0 < y < 1 \ \Rightarrow X$ and Y are SI. **T or F**
D) If X and Y are Gaussian then $E[XY]=E[X]E[Y]$. **T or F**

Question 18. In a telecommunication network, the transmitted signal S and received signal R are related as $R = S + N$, where N is the noise in the channel modelled as a standard normal RV. It is given that S is either 10 Volt or −10 Volt, and both are transmitted with equal probability.

A) $E(S) = 0$. **T or F**
B) The mean of R given that $S = 10$ is the same as mean of R given that $S = -10$. **T or F**
C) By central limit theorem, PDF of R is approximately Gaussian. **T or F**

D) Sketch the PDF of R.



Question 19. Two mutually exclusive events A and B can be statistically independent.
A) True B) False C) if $P(A) = P(B) = 0.5$ D) if $P(A) > P(B)$.

ANS: B. Since mutually exclusive means that occurrence or non-occurrence of one influences the other.

Question 20. A fair dice is thrown 2,000 times. Define X = number of times a multiple of 3 shows up. $P(X = 500)$ is given by

A) $[2000!/(500!500!1000!](1/3)^{1500}(2/3)^{500}$
B) $[1000!/(500!500!](1/3)^{500}(2/3)^{500}$
C) $[2000!/(500!1500!](1/3)^{500}(2/3)^{1500}$
D) $[2000!/(1000!1000!](1/3)^{1000}(2/3)^{1000}$

ANS: C. X is binomial with $n = 2{,}000$ and $p = 1/3$.

Question 21.
A) A sample of size one is a biased estimator of the population mean. **T or F**
B) Any estimator can be made unbiased. **T or F**
C) A consistent estimator is also an efficient estimator but the reverse is in general not true. **T or F**
D) $\frac{1}{n-1}(X_1 + X_2 + \ldots X_n)$ is an biased and consistent estimator. **T or F**

Question 22.
A) A WSS process is also SSS if its mean is time-invariant. **T or F**
B) Except for a Gaussian random process, an SSS process is also WSS. **T or F**
C) The variance of a zero mean WSS process is equal to the autocorrelation function at $\tau = 0$. **T or F**

D) If $\mathbf{X}(t) = A\cos wt + B\sin wt$ is a WSS process, where A and B are random variables, then the mean of A and B must be zero. **T or F**

Question 23.

A) If $\mathbf{X}(t)$ is constant, then $R_X(t)$ is also constant. **T or F**

B) For two jointly WSS random processes $\mathbf{X}(t)$ and $\mathbf{Y}(t)$, $R_{YX}(\tau) = -R_{XY}(-\tau)$. **T or F**

C) If $\mathbf{X}(t)$ and $\mathbf{Y}(t)$ are independent WSS random processes with mean μ_X and μ_Y, respectively, then $R_{XY}(t, t+\tau) = \mu_X \mu_Y$. **T or F**

D) Let $\mathbf{X}(t)$ be the input and $\mathbf{Y}(t)$ the output of a linear time-invariant system. A zero mean $\mathbf{X}(t)$ can produce a non zero mean $\mathbf{Y}(t)$. **T or F**

Question 24.

A) $S_X(\omega)$ is real and can be negative or positive depending on the process $\mathbf{X}(t)$. **T or F**

B) $S_X(\omega) = S_X(-\omega)$. **T or F**

C) There exists a process $\mathbf{X}(t)$ with power spectral density $S_X(\omega) = \sin\omega / \omega$. **T or F**

D) Show that the variance of a zero mean WSS process is equal to $\frac{1}{2\pi}\int_{-\infty}^{+\infty} S_x(\omega)d\omega$.

ANS: Since $R_X(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_x(\omega)e^{j\omega\tau}d\omega$ and $R_X(0)=Var[X(t)]$ if $X(t)$ is zero mean, we obtain $Var[X(t)] = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_x(\omega)d\omega$.

Question 25.

In a biomedical lab, a drug has a mean cure rate of 0.4. It was tested on 500 mice with a certain disease. Based on hypothesis testing at 0.05 level of significance, it is now claimed that the mean cure rate $\mu > 0.4$.

A) The hypotheses are H_0: $\mu = 0.4$ versus H_1: $\mu > 0.4$. **T or F**

B) The hypotheses are H_0: $\mu = 0.4$ versus H_1: $\mu < 0.4$. **T or F**

C) Maximum likelihood estimate of mean cure rate is given by $\frac{1}{n}(X_1 + X_2 + \ldots X_n)$. **T or F**

D) Maximum likelihood estimate of mean cure rate is always an efficient estimator. **T or F**

Question 26.

A) Type I error occurs when the alternative hypothesis is accepted when it is invalid. **T or F**

B) Type I error occurs when the alternative hypothesis is rejected when it is valid. **T or F**

C) Type II error occurs when the alternative hypothesis is rejected when it is valid. **T or F**

D) One can reduce Type I and Type II errors simultaneously. **T or F**

Question 27.

A) In interval estimation, the larger the sample size, the larger the width of the interval would be, for a fixed level of confidence. **T or F**

B) A preference test based on interval estimation is used to establish whether Singaporeans prefer chicken rice or laksa. Based on a sample of size 1000, it is found that the proportion of Singaporeans preferring chicken rice lies in the interval (0.2, 0.3). Therefore, we can conclude that Singaporeans prefer laksa. **T or F**

C) In interval estimation, the larger the level of confidence, the larger the width of the interval would be, for a fixed sample size. **T or F**

D) Derive the smallest sample size required for interval estimation based on Z under level of significance α and margin of error $\pm E$.

ANS: From $\bar{X} - \frac{\sigma}{\sqrt{n}} z_{\alpha/2} < \mu < \bar{X} + \frac{\sigma}{\sqrt{n}} z_{\alpha/2}$, we obtain $E = \frac{\sigma}{\sqrt{n}} z_{\alpha/2}$. Thus, the smallest sample size needed is $\left[\frac{\sigma_{\max}}{E} z_{\alpha/2}\right]^2$, where $\sigma_{\max}$ is the maximum possible value of σ.

Question 28.

Given a random variable X uniformly distributed over $[-1, 2]$, let $Y = X^2$. $P(Y < 3)$ is

A) close to 0.6
B) close to 0.7
C) close to 0.8
D) close to 0.9

ANS: $P[Y < 3] = P[X^2 < 3] = P[-\sqrt{3} < X < \sqrt{3}] = P[-1 < X < \sqrt{3}] = (\sqrt{3} + 1)/3 \approx 0.9$.

Question 29.

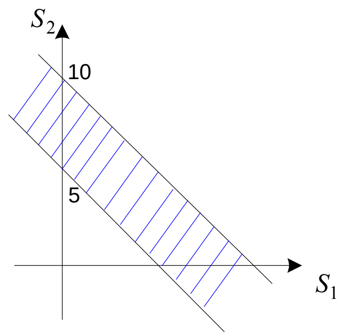
Let $S_1 \sim N(\mu_1, \sigma_1^2)$ and $S_2 \sim N(\mu_2, \sigma_2^2)$ be two signals having Gaussian PDF. Assume that S_1 and S_2 are uncorrelated.

A) $S_1 + S_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$ **T or F**

B) $S_1 + S_1 + S_1 + S_1 + S_1 \sim N(5\mu_1, 25\sigma_1^2)$ **T or F**

C) S_1 and S_2 are SI. **T or F**

D) Mark the region corresponding to the event A: $5 < S_1 + S_2 < 10$.



(5 marks)

Question 30.

A manufacturing process is modeled by a binomial RV X with parameters $(N, 0.3)$. Another process is modeled by a binomial RV Y with parameters $(M, 0.3)$. Assume X and Y are independent.

A) $X+Y$ is also a binomial random variable. **T or F**

B) $X+Y$ has $N+M$ Bernoulli trials. **T or F**

C) If M is sufficiently large, then $X + Y$ can be approximated by a Gaussian random variable. **T or F**

D) Write an exact expression for P($X + Y < 10$).

$$P(X+Y<10)=\sum_{k=0}^{9}\binom{N+M}{k}0.3^k 0.7^{N+M-k}$$

Question 31.

The number of data packets arriving at a node A is modeled by Poisson RV X:

$$P(X=k)=e^{-10{,}000}10{,}000^k/k!,\ \ k=0,1,\ldots$$

A) $P(X < 20)$ is close to 0. **T or F**

B) $P(X > 5)$ is close to 1. **T or F**

C) We can find P($X > 500$) by approximating X by a Gaussian random variable. **T or F**

D) A gets congested if X exceeds 8000. Write an expression for P(A is congested). Describe how the expression can be evaluated.

ANS: $P(X>8000)=1-P(X\le 8000)=1-\sum_{k=0}^{8000}e^{-10{,}000}10{,}000^k/k!$

Approximate X by Gaussian, and then use the standard Gaussian table to find the probability.

Question 32.

A series of N uncorrelated signals from a satellite are represented by the Gaussian random variables $X_1, X_2, \ldots X_N$.

A) $X_1+X_2+\ldots+X_N$ is Gaussian . **T or F**

B) $X_1, X_2, \ldots X_N$ are SI. **T or F**

C) The joint PDF of $X_1, X_2, \ldots X_N$ is the convolution of the individual marginal PDFs of $X_1, X_2, \ldots X_N$. **T or F**

D) Find the PDF of $X_1 - 2X_2 + 3X_3 + \ldots + (-1)^{N-1} NX_N$.

ANS: Let $X_i \sim N(\mu_i, \sigma_i^2)$ for $i = 1,2,\ldots,N$.

$X_1, X_2, \ldots X_N$ are uncorrelated $\Rightarrow$ $X_1, X_2, \ldots X_N$ are statistically independent

$\Rightarrow$ $X_1 - 2X_2 + 3X_3 + \ldots + (-1)^{N-1} NX_N \sim N(\mu, \sigma^2)$, where

$\mu = \mu_1 - 2\mu_2 + 3\mu_3 + \ldots (-1)^{N-1} N\mu_N$ and $\sigma^2 = \sigma_1^2 + 4\sigma_2^2 + 9\sigma_3^2 \ldots + N^2\sigma_N^2$.

Therefore, PDF of is $X_1 - X_2 + \ldots + (-1)^{N-1} X_N$ is $\dfrac{1}{\sqrt{2\pi}\sigma} \exp\left[-\dfrac{(x-\mu)^2}{2\sigma^2}\right]$.

Question 33.

A new cancer drug will cause side effect X with probability 0.2, side effect Y with probability 0.4, and both side effects X and Y with probability 0.06.

A) The side effect X and side effect Y are independent. **T or F**
B) The side effect X and side effect Y are mutually exclusive. **T or F**
C) Can't tell whether the side effects X and Y are independent. **T or F**
D) Can't tell whether the side effects X and Y are mutually exclusive. **T or F**

Question 34.

Let X be a binomial RV with parameter N and p.

A) X is make up of N Bernoulli trials **T or F**
B) X is make up of non SI Bernoulli trials. **T or F**
C) When N becomes very large and p becomes very small such than Np is constant, then the binomial RV tends to a negative binomial RV. **T or F**
D) When N becomes sufficiently large X tends to a Gaussian RV. **T or F**

Question 35.

A) $E[X]$ and V[X] completely identify the RV X. **T or F**
B) MGF of X completely identify RV X. **T or F**
C) PDF of X completely identify the RV X. **T or F**
D) CDF of X completely identify the RV X. **T or F**

Question 36.

Let X be a Gaussian random variable with mean 0.5. If $P(0.1 < X < 0.5) = 0.3$ and $P(0.5 < X < 1.0) = 0.4$, then

A) $P(0.1 < X < 0.8) < 0.7$ **T or F**

B) $P(X < 1.0) = 0.9$ **T or F**
C) $P(X > 0.1) = 0.8$ **T or F**
D) P(X>0.9)>0.5 **T or F**

Question 37.
Let X and Y be two orthogonal RV.

A) The mean of X and mean of Y must be zero. **T or F**
B) If mean of X and mean of Y are zero, then X and Y are SI. **T or F**
C) X and Y must also be SI. **T or F**
D) If X and Y are jointly Gaussian, they are also SI. **T or F**

Question 38.
Let X and Y be two RV with marginal PDFs $f(x)$ and $h(y)$, respectively. Their joint PDF is denoted by $f(x, y)$.

A) If X and Y are independent, then $E[X^2Y^4] = E[X^2]E[Y^4]$ **T or F**
B) If $E[XY] = E[X]E[Y]$ and X and Y are Gaussian RV, then X and Y are independent. **T or F**
C) Only if X and Y are uncorrelated Gaussian RV we have $E[XY] = E[X]E[Y]$ **T or F**
D) If X and Y are independent, then write the PDF of $X + |Y|$ as a convolution of two PDFs.

ANS: X and Y are independent $\Rightarrow$ X and $|Y|$ are also independent.
Let $Z=|Y|$ and PDF of Z be $g(z)$. Then PDF of $W=X+|Y|$ is the convolution of $f(x)$ and $g(z)$ given as

$$\int_{-\infty}^{\infty} f(u)g(w-u)du.$$

Question 39.
Consider a three-face coin. The face and their probabilities of occurrence at any given toss are given by

Success: probability=r
Failure: probability=s
Don't care: probability=t

It is clear that $r + s + t=1$. This coin is tossed N times. Define a two dimensional RV as

(X, Y)=(total number of successes, total number of failures).

A) Are these Bernoulli trials?

ANS: No. They require only two possible outcomes for each trial. We have three.

B) Write the 2-dimensional joint PDF of (X, Y).

ANS: $P(X=a,Y=b)=\dfrac{N!}{a!b!(N-a-b)!}r^a s^b t^{N-a-b}.P(X=k,Y=5)$

This is defined for all values of a and b such that $a\geq 0, b\geq 0$ and $a+b\leq N$. The derivation can be done along the same lines as the derivation for binomial PDF.

C) Write the PDF of the random variable Z define as $Z=X+Y$.

ANS: $Z=X+Y$ means we have either (success and failure) as one outcome or don't care as the other outcome. P(success or failure in one trial)$=r+s$. Thus Z is binomial with parameters N and $r+s$.

D) Write the marginal PDF of X.

ANS: It is clear that $X=a$ means that we are looking for a successes in N trials. This is binomial PDF with parameters N and r.

E) Write the conditional PDF of X given that $Y=5$.

ANS: $$\begin{aligned}P(X=k\mid Y=5)&=\frac{P(X=k,Y=5)}{P(Y=5)}\\&=\left[\frac{N!}{k!5!(N-5-k)!}r^k s^5 t^{N-5-k}\right]\div\left[\frac{N!}{5!(N-5)!}s^5(1-s)^{N-5}\right]\\&=\frac{(N-5)!}{(N-5-k)!k!}\left(\frac{r}{1-s}\right)^k\left(\frac{t}{1-s}\right)^{N-5-k}\\&=\frac{(N-5)!}{(N-5-k)!k!}\left(\frac{r}{r+t}\right)^k\left(\frac{t}{r+t}\right)^{N-5-k},\end{aligned}$$

which is a binomial PDF with parameters $N-5$ and $r/(r+t)$.

F) Are X and Y SI.

ANS: No. This is clear from the form of $P(X=a, Y=b)$ as stated in (B) above.

Question 40.

When P implies Q and Q implies P, we write $P \Leftrightarrow Q$. Let X and Y be RVs having marginal PDFs $f_X(x)$ and $f_Y(y)$, respectively and joint PDF $f_{X,Y}(x,y)$.

A) $f_{X,Y}(x,y)=f_X(x)\,f_Y(y) \Leftrightarrow E[XY]=E[X]E[Y]$ **T or F**

B) X, Y are SI $\Leftrightarrow$ X, Y are uncorrelated **T or F**

C) X, Y are Gaussian RVs, $f_{X,Y}(x,y)=f_X(x)\,f_Y(y) \Leftrightarrow \text{cov}(X,Y)=0$ **T or F**

D) $E[X]=0$ or $E[Y]=0$, X, Y are uncorrelated $\Leftrightarrow$ X, Y are orthogonal **T or F**